Nonlinear analysis of a multi-sloshing mode Tuned Liquid Sloshing Damper coupled with a tall building under across-wind loading

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SUMMARY:
This paper presents an incremental formula for nonlinear time domain analysis of multi-sloshing mode Tuned Liquid Sloshing Damper (TLSD). Nonlinear screen damping term of TLSD is implemented without any simplification such as linearization of the term as in other studies; and multiple fundamental sloshing modes of the water are also implemented whereas only one sloshing mode is considered in previous studies. The nonlinear TLSD formula is compared with dynamic test of the water tank. The interaction between the building's motion and self-excited wind force is also included in terms of aerodynamic damping and stiffness as functions of reduced frequency. For the time-domain analysis, the frequency-dependent aerodynamic damping and stiffness are represented by constant coefficients and additional supplemental equations which are solvable in time-domain. All the terms including the nonlinear terms are expressed in incremental formula based on time integration in combination with the linearization of the quadratic terms. Dynamic behaviour of a generic 300m-tall building with square floor plan equipped with a TLSD on top are also analysed under strong across-wind loading excitation. In the example, the effects of nonlinear water damping, multi-sloshing modes of TLSD, and aerodynamic damping terms are investigated in detail.

Keywords: nonlinear time domain analysis, Tuned Liquid Sloshing Damper, wind loading on tall buildings

1. INTRODUCTION

To mitigate excessive wind-induced motions of tall buildings, TLSDs have been frequently used because of their low cost, simple frequency tuning, and low maintenance (Kareem, 1987). The sloshing motion of the liquid in a tank can be expressed as a combination of infinite sloshing modes based on potential flow theory (Baucer, 1984). In a TLSD, the building's dynamic energy is dissipated through porous screens (Warnitchai, 1997) in terms of a damping force. The damping force created by the water is a quadratic term of water velocity, which introduces nonlinearity in modelling the damping force in the analysis.

In previous studies (Caughey, 1963; Tait, 2008), the analysis of TLSD has been simplified by applying equivalent linearization method to the damping force water velocity term. Generally, contribution of higher sloshing modes has also been ignored. However, the nonlinearity and the higher modes become more important for the tanks with longer natural periods and during transient state of water sloshing motion. In this paper, the nonlinearity of water damping terms and the higher modes are considered.

2. TIME DOMAIN ACROSS-WIND BUILDING MOTION COUPLED WITH TLSD

Fig.1(a) illustrates a schematic diagram of a building coupled with multi-sloshing mode TLSD, modeled as multi-degree of freedom system (Fig.1(b)). In equation form, the dynamic motion of a tall building coupled with a tuned liquid sloshing damper, accounting for aerodynamic damping and stiffness, in the time domain, is expressed as follows. As shown in Eq. (2), the equation of motion for the TLSD is a nonlinear equation due to the velocity-dependent nonlinear liquid damping term, which is the third term in the left side of the equation.

$$\left(1 + \frac{\rho_w bhL}{\tilde{m}_1}\right)\ddot{\tilde{x}} + \sum_{n=1}^{n_d} \frac{m_{eq,n}}{\tilde{m}_1}\ddot{x}_{r,n} + 2\zeta_1\omega_1\dot{\tilde{x}} + \omega_1^2\tilde{x} = \frac{k_m}{\tilde{m}_1}M - 2\eta\omega^2(\alpha + i\beta)\tilde{x} \tag{1}$$

$$\ddot{\tilde{x}} + \ddot{x}_{r,n} + \frac{2(1-cos(n\pi))}{n\pi L} tanh^2\left(\frac{n\pi h}{L}\right) C_l\varDelta_n\varXi_n|\dot{x}_{r,n}|\dot{x}_{r,n} + \omega_{eq,n}^2 x_{r,n} = 0 \tag{2}$$

In the above: $\tilde{x}$, generalized coordinate of a building structure; $x_{r,n}$, relative motion of liquid sloshing mode n, with respect to $\tilde{x}$; $\tilde{m}_1$, ζ_1 ω_1, building's modal mass, damping ratio and natural angular frequency, respectively; k_m, mode shape correction factor; M, base moment; $\alpha, \beta=$ experimental frequency-dependent aerodynamic stiffness and damping; $i = \sqrt{-1}$; ρ_w, liquid density; $h, b,$ and L, water depth, tank width and length, respectively ; n_d, total sloshing modes; $m_{eq,n}$, $\omega_{eq,n}$, the effective mass and natural angular frequency of liquid sloshing mode n, respectively; C_l, loss coefficient of a screen; $\Delta_n = 0.33 + \sinh^{-2}(n\pi h/L)$; $\Xi_n = \sum_{j=1}^{ns} \sin^3(n\pi x_j/L)$; x_j, x-coordinate of screen j; ns, number of screens.

After converting the frequency-dependent aerodynamic terms of α and β in Eq. (1) into constant coefficient terms based on Rational Function Approximation (Fujino et al., 1995), the (converted) equations of motion are solved by expressing the time derivatives of the time-dependent variables in incremental form at time, $t + \Delta t$, and iteration, i, based on the Newmark time integration scheme (Bathe, 1982). The nonlinear term in Eq. (2) can be linearized by using the value evaluated at previous step $(n-1)$, as follows in Eq. (3) based on successive substitution-type iteration method (Jeong et al, 2002).

$$\ddot{\tilde{x}}_i^{t+\Delta t} + \ddot{x}_{r,n,i}^{t+\Delta t} + C_{eq,n}|\dot{x}_{r,n,(i-1)}^{t+\Delta t}|\dot{x}_{r,n,i}^{t+\Delta t} + \omega_{eq}^2 x_{r,n,i}^{t+\Delta t} \approx 0 \tag{3}$$

3. EXAMPLES

3.1 Example 1: Dynamic Testing and Validation of Nonlinear Formula

To verify the proposed formula, a 1/10-scale TLSD is analyzed and compared with dynamic test results performed on a dynamic test rig. The dynamic test rig is a pendulum weighing 4,330 kg connected to a set of springs, having a system stiffness of 6.94×10^4 N/m. A tank measuring 1.3 m (L) by 0.37 m (b) is filled with 0.3 m depth of water (h) to generate a sloshing motion of 3.83 rad/s ($\omega_{eq,1}$). Two screens, each of 55% solidity are placed at $x_j/L =$ 0.4 and 0.6, with loss coefficient (C_l) estimated to be 5.44. The pendulum is subject to a sinusoidal excitation with free decay. The damping coefficient of the pendulum is equal to 0.5%.

Numerically, the total non-conservative non-linear damping force acting on the screen by the sloshing water for ns modes can be determined from the sum of the third term in Eq. (2), from n = 1 to ns (Tait, 2008), using the computed relative displacement of the water for each mode, $\dot{x}_{r,n}$. In the "linear model", the non-linear velocity term of damping force is approximated as

$|\dot{x}_{r,n}|\dot{x}_{r,n} \cong \sqrt{8/\pi}\sigma_r\omega_{eq}\dot{x}_{r,1}$ for n = 1, where σ_r is the standard deviation of the relative water displacement. Since σ_r is not known *a priori,* it must be assumed given the estimated building response; σ_r will be taken as 0.021 m/s in this example.

By affixing a load cell to the upper portion of the screen, the TLSD damping force is measured during free decay. Fig. 2 below superimposes the damping force decay response from the experimental study (grey dots) to the linear analysis (blue) and six mode non-linear analysis (red). The proposed non-linear formula is shown to be valid and reasonable, given good phase and amplitude matching with the experimental results, in comparison with the linear formula results.

3.2 Example 2: Across-Wind Response of a Coupled Tall Building-TLSD System

In this example, a tall building of height 300 m, with plan dimensions 30 m × 30 m is analysed under mean hourly wind speed of 23.8 m/s defined at 10 m height above the grade in open exposure, exponential mean wind speeds with exponent of 0.14. The mass density of the building is 225 kg/m^3, with a structural damping ratio of 2%, and typical mode shape exponent of 1.5. The effect of aerodynamic damping is investigated under a turbulence intensity of 6%.

Since tall buildings with low natural frequencies experience excessive accelerations, a series of coupled building-TLSD systems are analysed numerically for structural natural frequencies ranging from 0.12 Hz to 0.16 Hz. Each TLSD is optimally tuned, given an effective mass ratio (= $m_{eq,1}/\widetilde{m}_1$) of 1.44%. The tank sizes are listed in Table 1, below. Two screens are placed at x_j/L = 0.4 and 0.6, having C_l = 5.05, for solidity, 55%. Time domain analysis is performed using the generated wind load time series derived from AIJ 2006 spectra.

Table 1 compares the acceleration response for the coupled system under the linear and non-linear time series analyses, with aerodynamic damping. Generally, good agreements are made between both the time series analysis and the spectral analysis. Higher discrepancies between the time domain methods occur as the natural frequency approaches 0.1 Hz, where negative aerodynamic damping for this building is strongest; in each case, the non-linear results are found to be more conservative for buildings having low natural frequencies. Since the non-linear methodology can determine of the damping term implicitly based on the water sloshing response, it is thus well suited for cases in which active hydrodynamic effects are present. Given that the linear model also requires an additional assumption (namely that of σ_r), the non-linear methodology provided in this paper provides an opportunity for more straightforward prediction of the nonlinear water damping effects.

Table 1 Comparison of cross-wind response between the non-linear and linear analysis methodologies of a coupled tall building-TLSD system with a range of building natural frequencies.

Tuned Frequency, f_T (Hz)	Tank Depth, b (m)	Tank Length, L (m)	Water depth, h (m)	RMS of Acceleration (milli-g)		Ratio, A_{NL}/A_L (%)
				Linear, A_L $\sigma_r = 0.12$	Non-Linear, A_{NL}	
0.12	6.880	19.0	2.16	18.79	23.41	+24.6
0.13	8.180	17.0	2.04	15.48	17.64	+14.0
0.14	7.711	16.5	2.26	13.17	14.85	+12.8
0.15	8.270	15.4	2.28	12.05	13.34	+10.7
0.16	8.022	14.9	2.47	10.35	11.17	+7.9

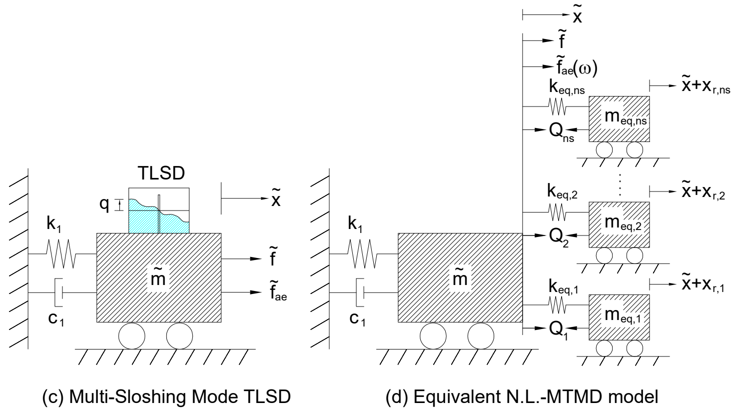

Figure 1 Schematic Diagram of Tall Building-TLSD System

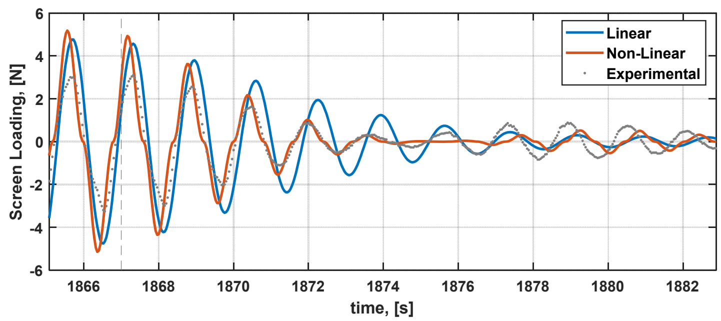

Figure 2 Experimental free decay vibration of a coupled pendulum-TLSD system compared to numerical results computed using linear and non-linear analysis methods

4. CONCLUSION

The nonlinear multi-sloshing mode formula represents accurate modelling of water sloshing modes in TLSD. The formula is applied to a TLSD-tall building coupled system in consideration of aerodynamic damping and the effects of each component of the system is investigated in detail.

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